Temporal backward planar projection of acoustic transients

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(Received 30 September 1997; accepted for publication 22 December 1997)

Backward projection schemes use data in front of a transmitter to reconstruct a field at closer points. Existing techniques have concentrated propagating radial and temporal information along constant planar cross sections in front of a planar source. This approach requires a careful consideration of evanescent waves, as the transfer function used to backproject in space causes evanescent wave solutions to increase exponentially with the projected distance. Erroneous signals may result from exponentially increasing noise, experimental error or roundoff error. A method is presented that is designed to work with imaging methods that record three-dimensional spatial data at constant times. Several widely used optical methods are of this type. Our algorithm projects the field backward in time via linear wave theory. The approach is similar to previously reported methods but is designed to work with time-constant data and avoids problems associated with evanescent waves. © *1998 Acoustical Society of America.* [S0001-4966(98)01504-5]

PACS numbers: 43.20.Bi [ANN]

INTRODUCTION

Linear planar propagation methods may be used for full field characterization in space-time using limited data. These numerical methods generally operate in the wave vector frequency domain using Fourier transforms[1] or Hankel transforms.[2,3] A wave vector, time-domain approach[4] has also been described. These methods use field information along a planar cross-section at a constant distance z_0 from a baffled planar source. The signal may be forward projected away from the source or backprojected toward the source. Backward projections of experimentally measured harmonic pressure amplitudes,[5,6] $P(\mathbf{r},z_0)$ have been reported. Recently, experimental time dependent transient fields, $p(\mathbf{r},z_0,t)$,[7,8] were forward projected.

Backward projection of time-dependent transients, however, is more complicated. Fleischer and Axelrad[9] have addressed problems associated with exponential growth of evanescent wave solutions. Williams and Maynard[5] used data very close to the source to avoid complications. Filtering methods[6] and approximation techniques[10] have also been used to partially amend this problem.

An alternative to projecting the field backward in space is to record spatial field information and time reverse the field. To provide distinction from the recent work of Fink *et al.*[11,12] we refer to this technique as temporal backward projection. Fink's work concentrates on the use of acoustic time-reversal mirrors. These mirrors physically reproject the conjugate of their received signals, focusing the field back on the original source. Through an iterative process a strong target in a multitarget media may be singled out. Prada *et al.*[13] provide an elegant analysis of this process. In contrast, the temporal backward projection described here assumes full knowledge of the field in a medium and is designed to reproduce acoustic signal information at arbitrary points at earlier times. A single measurement at a given time allows the full spatial field to be reconstructed at any earlier time.

This temporal approach is well suited for imaging systems which record field data at constant times. Examples of these systems include tomographically reconstructed optical pulsed schlieren, interferometry, and holographic techniques. The transfer function employed in this temporal projection method is mathematically similar to existing techniques, however it exhibits advantages over spatial backward projections. The transfer function does not display the exponential growth associated with spatial projections. In addition, the time constant imaging method is capable of recording complete field information, while spatial projection methods record temporal and radial data over a constant plane and ultimately must truncate the field.

We present a practical algorithm for temporal forward/backward projection. Axisymmetric sources are assumed, allowing Hankel transform techniques to be used. The underlying theory is discussed in Sec. II. An impulse response approach[14] is used to generate numerical solutions to the field resulting from a Gaussian pulse input into a baffled circular radiator. Data obtained at a specific time, t_0 is projected backward to a new time t and compared with direct numerical calculations. To demonstrate the range of the method, we select a starting acoustic pulse far from the source. The back-projection is shown to reconstruct major field features at earlier times. Results and procedures are outlined and discussed in Sec. III.

I. THEORY

A general pressure field in Cartesian coordinates may be written in terms of its three dimensional Fourier transform

$$p(\mathbf{r},t)=\frac{1}{(2\pi)^{3/2}}\int\int_{-\infty}^{\infty}\int P(\mathbf{K},t)e^{i\mathbf{K}\cdot r}\,d^3\mathbf{K}, \tag{1}$$

where the wavenumber $K^2=k_x^2+k_y^2+k_z^2$. It is assumed that this function satisfies the linear wave equation

$$\nabla^2 p(\mathbf{r},t)=\frac{1}{c^2}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2}, \tag{2}$$

with sound velocity c. Substitution of the right hand side of Eq. (1) into Eq. (2) yields a Helmholtz equation,

$$\frac{d^2}{dt^2}P(\mathbf{K},t)+c^2K^2P(\mathbf{K},t)=0. \tag{3}$$

The solutions to this second-order equation describe the advanced and retarded propagation, respectively. The advanced solution is used for the projection problem:

$$P(\mathbf{K},t)=P(\mathbf{K},t_0)e^{-ic(t-t_0)K}, \tag{4}$$

where the exponential term is the transfer function associated with the projection. The field $P(K,t_0)$ is obtained by direct measurement of the field at t_0. It is noted that the exponent of the transfer function is pure imaginary, thus eliminating the exponentially increasing signals associated with evanescent waves.

The pressure fields considered in this paper are axisymmetric, thus a cylindrical coordinate system is more natural for describing the projections. The radiator is assumed to be a circular disk in the $y-z$ plane and centered about the z-axis. A change of variables is made so that $k_x=k_\rho\cos\phi$, $k_y=k_\rho\sin\phi$, $K=\sqrt{k_\rho^2+k_z^2}$, and the field is independent of the polar angle ϕ. The pressure field in wave-vector space is now represented by

$$P(k_\rho,k_z,t)=P(k_\rho,k_z,t_0)e^{-ic(t-t_0)\sqrt{k_\rho^2+k_z^2}}. \tag{5}$$

To reconstruct the field at the time t, the zeroth-order Hankel transform pair is introduced:

$$P(k_\rho,k_z,t)=\int_0^\infty p(\rho,k_z,t)J_0(k_\rho\rho)\rho\ d\rho, \tag{6}$$

$$p(\rho,k_z,t)=\int_0^\infty P(k_\rho,k_z,t)J_0(k_\rho\rho)k_\rho\ dk_\rho. \tag{7}$$

The zeroth-order Bessel function is given by $J_0(k_\rho\rho)$. Using the conjugate transform of Eq. (1) and Eq. (6), the field may be expressed as

$$p(\rho,z,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\int_0^\infty P(k_\rho,k_z,t_0) \times e^{-ic(t-t_0)\sqrt{k_\rho^2+k_z^2}}J_0(k_\rho\rho)e^{ik_z z}k_\rho\ dk_\rho\ dk_z. \tag{8}$$

The pressure $P(k_\rho,k_z,t_0)$ is obtained by direct measurement of the field at t_0.

II. PROCEDURES AND RESULTS

The impulse response method is used to simulate acoustic fields from an underwater ultrasonic source. A Gaussian input signal, full width at half-maximum of 1 μs, is considered. The radiator is assumed to be a 2.8-cm-diam baffled circular disk.

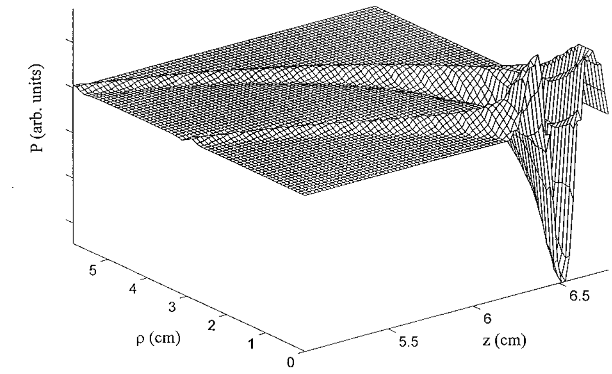

FIG. 1. Acoustic signal at 30 μs.

A simulated field is calculated for the time $t_0=30$ μs after the start of the launching of the Gaussian pulse. This field is then propagated backward in time using a matrix based algorithm to calculate a discrete approximation to Eq. (8). The field is represented by a 90×150 matrix describing the ρ and z dimensions, respectively. Specifically, fields reconstructed at 15, 9, and 3 μs are presented. The projected fields are compared with direct impulse response calculations over a series of time slices. The starting pressure field at 30 μs is shown in Fig. 1 as a function of its spatial coordinates, ρ and z. The field is measured in the region $0\leqslant\rho\leqslant6$ cm and $5\leqslant z\leqslant6.8$ cm, and is assumed zero elsewhere. This field displays the characteristic shape of a Gaussian signal far from the source. The impulse response field calculation at 3 μs is shown in Fig. 2(a). This signal is compared with the reconstructed back-projection from 30 μs to 3 μs, in Fig. 3(a). It is noted that the backprojection carries additional small oscillations along both the radial and propagation directions. The projection algorithm is, of course, discrete and band limited and ultimately cannot completely reconstruct the surface describing the field. The signal at 9 μs is shown in Fig. 2(b) and its backprojection from 30 μs in Fig. 3(b). Direct comparison reveals the backprojection's ability to reconstruct the half-ring shaped waveform centered about the transducer edge. At 15 μs the field now exhibits two local minima on-axis; the peak at 2.4 μs being from the main signal and the second a result of the acoustic ''edge wave.'' This feature, shown in Fig. 2(c), is reproduced in Fig. 3(c), the projection from 30 μs to 15 μs. This projection, however, reproduces a slightly broader edge wave than the direct numerical calculation predicts. Once again, this seems to result from the band limitations set by the numerical algorithm.

Each projection took under one minute to compute using a 133-MHz Pentium processor. The spatial sampling is 16 cm^{-1} in the radial direction and 80 cm^{-1} along z for each of the fields in Fig. 3. The spatial-frequency sampling used in the algorithm is 21 cm over $0\leqslant k_z\leqslant60$ cm^{-1} and 50 cm over $0\leqslant k_\rho\leqslant20$ cm^{-1}.

III. SUMMARY

The temporal backward projection algorithm is shown to accurately reconstruct major features of the pressure field

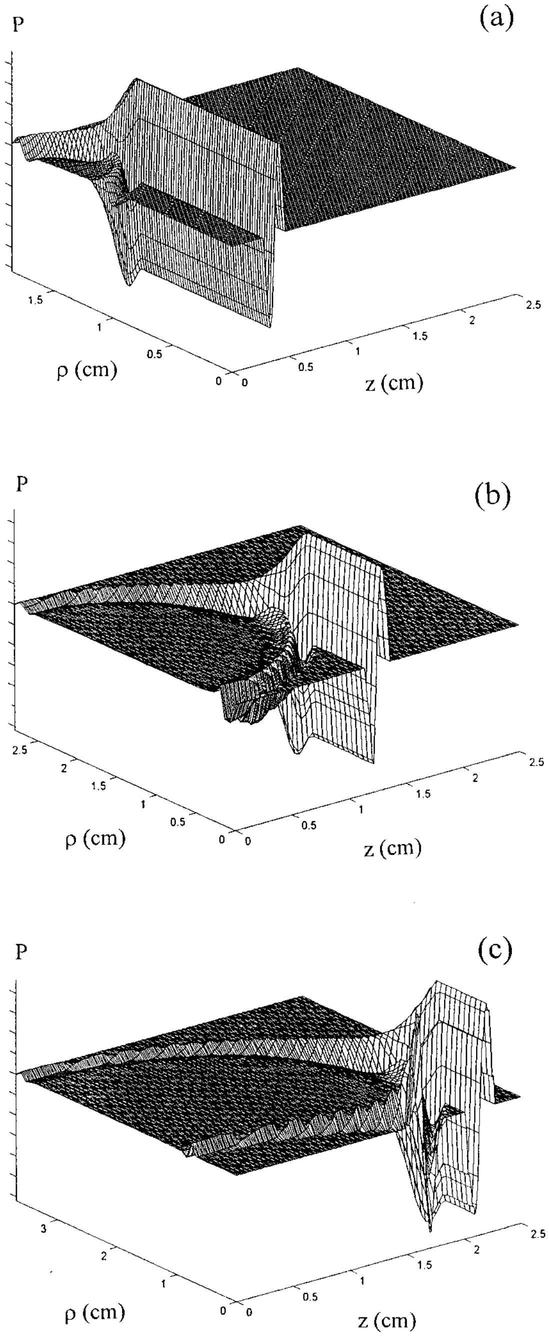

FIG. 2. Impulse response calculation of pressure field **P** in arbitrary units at (a) $t=3$ μs, (b) $t=9$ μs, and (c) $t=15$ μs.

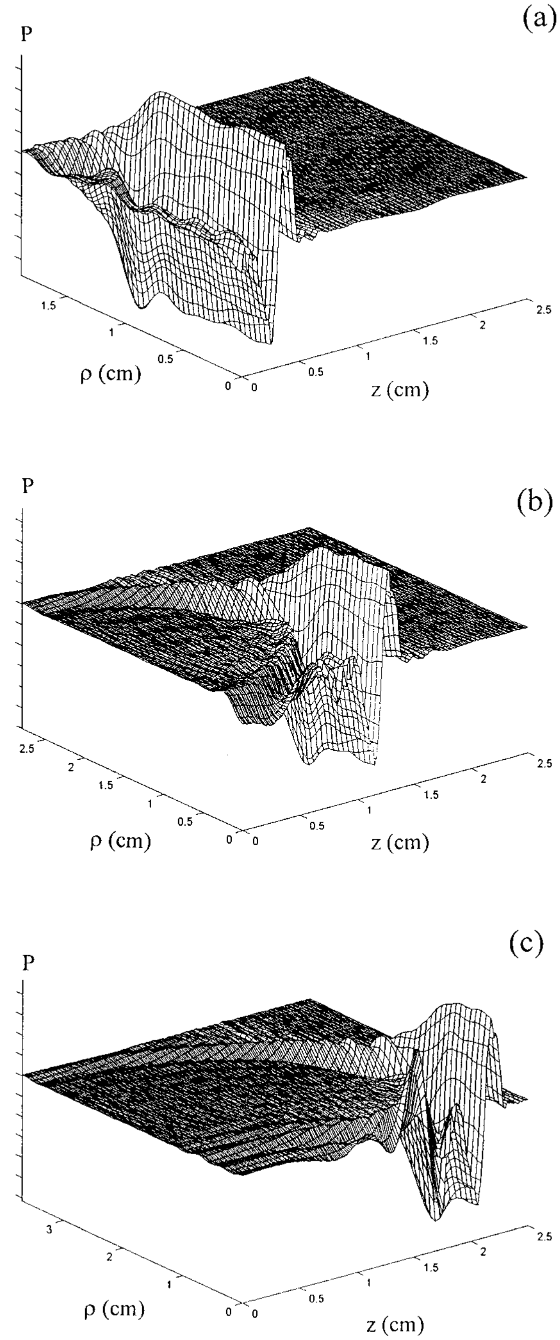

FIG. 3. Backward projection of pressure field from $t_0=30$ μs to (a) $t=3$ μs, (b) $t=9$ μs, and (c) $t=15$ μs.

resulting from an axisymmetric Gaussian input signal. Failure of the Hankel transform based algorithm to reconstruct sharp features in the field near the source seems to result from band limitations along both the radial and propagation axes. In practice, fields measured experimentally tend to be smooth and in this respect may prove easier to reconstruct than synthetic data. There are advantages to the reported method over previously reported spatial projection techniques. Field imaging systems, such as the modulated schlieren apparatus, which record spatial field information at a specific time, may more readily use the temporal method for projecting fields. Further the transfer function implemented in the temporal backward projection method does not exhibit the complications associated with evanescent waves. Finally, since the field as a specific time is spatially finite for pulsed signals, an entire field may be imaged where previous methods require truncation. Further study will concentrate on the backward projection of experimental transient signals. In addition, a numerical investigation will be conducted concerning the effects of noise on the algorithm.

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